Spiral cylindrique sans courbes terminales

Anisochronisme causé par un décentrage initial du spiral

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm$

Elinvar $\quad \rho_s = 8\times10^3\ kg\cdot m^{-3} \quad E = 1.7\times10^{11}\,Pa \quad G = 6.538\times10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654\times10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

Spiral non déformé en position de repos (approximation de Haag)

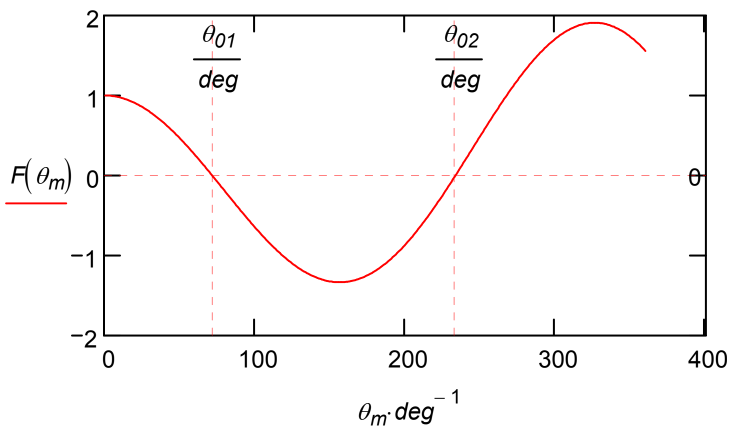
$\theta_0 = 270\,deg \quad F(\theta_0) := J0(\theta_0) - \theta_0\cdot J1(\theta_0) \quad \Delta(\theta_0) := \frac{2}{\psi_0^{\,2}}\cdot\left(-1 + F(\theta_0)\cdot\cos(\psi_0)\right) \quad \Delta(\theta_0) = -1.849\times10^{-4}$

$\mu(\theta_0) := -86400\cdot\Delta(\theta_0) \quad \boxed{\mu(\theta_0) = 15.98} \quad \boxed{\mu(220\cdot deg) = 52.235}$

$x := 100\cdot deg \quad \theta_{01} := racine(F(x), x) \quad \theta_{01} = 72\,deg \quad \theta_{m1} := racine\left(\frac{d}{dx}F(x), x\right) \quad \theta_{m1} = 156.7\,deg$

$x := 300\cdot deg \quad \theta_{02} := racine(F(x), x) \quad \theta_{02} = 233.7\,deg \quad \theta_{m2} := racine\left(\frac{d}{dx}F(x), x\right) \quad \theta_{m2} = 326.1\,deg$

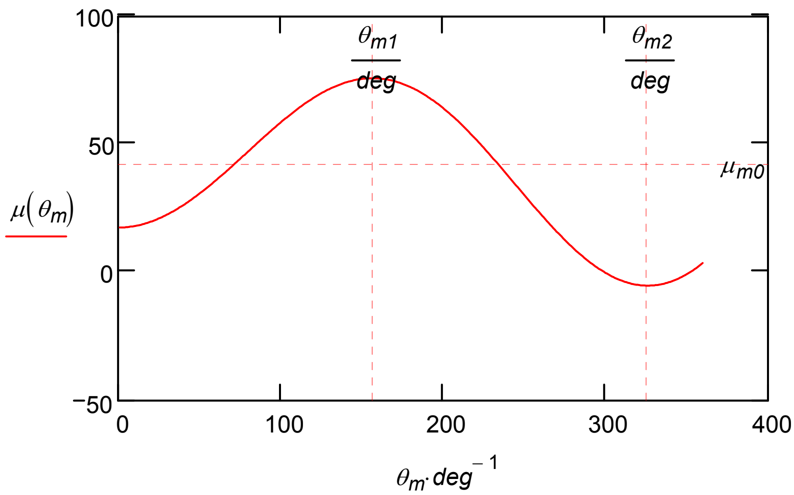
$\theta_m := 1\cdot deg, 2\cdot deg .. 360\cdot deg$



$\mu_{m0} := \frac{86400\cdot 2}{\psi_0^{\,2}}$

$\mu_{m1} := -86400\cdot\Delta(\theta_{m1})$

$\mu_{m1} = 75.845$



$\mu_{m2} := -86400\cdot\Delta(\theta_{m2})$

$\mu_{m2} = -5.153$

Spiral déformé en position de repos par décentrage de la virole

$h_{déc} := 0.2 \cdot mm \qquad \beta_{déc} := 20 \cdot deg \qquad \sigma 2 := R_0^{\,2}$

$$X_1(\theta) := \frac{2}{\sigma 2} \cdot \left(\frac{\theta}{\psi_0 + \theta} \cdot R_0 \right)^2 \cdot \left(1 - \cos\left(\psi_0 + \theta\right)\right) \qquad \gamma_1(\theta) := \frac{d}{d\theta} X_1(\theta)$$

$$\delta_1(\theta_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2 \cdot \pi} \gamma_1\left(\theta_0 \cdot \cos(\varphi)\right) \cdot \cos(\varphi)\, d\varphi \qquad \mu_1(\theta_0) := -86400 \cdot \delta_1(\theta_0) \qquad \boxed{\mu_1(\theta_0) = 24.581}$$

$$\delta_3(\theta_0, h, \beta) := \frac{2 \cdot h}{\psi_0 \cdot L} \cdot \left(\cos(\psi_0 - \beta) + \cos(\beta)\right) \cdot \left(1 - F(\theta_0)\right) - \frac{2 \cdot h}{L} \cdot \left(\sin(\psi_0 - \beta) + \sin(\beta)\right) \cdot J0(\theta_0)$$

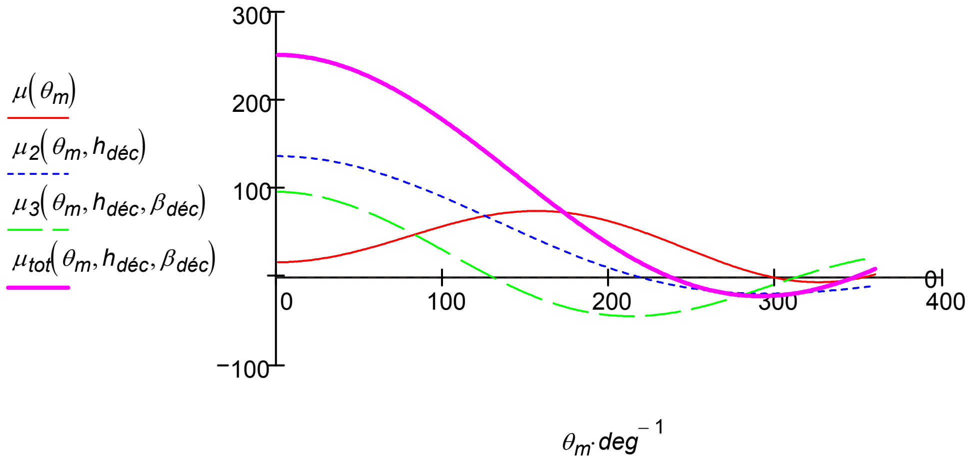
$$\delta_2(\theta_0, h) := \frac{-2 \cdot h^2}{R_0^{\,2}} \cdot \frac{J1(\theta_0)}{\theta_0} \qquad \delta_{tot}(\theta_0, h, \beta) := \delta_1(\theta_0) + \delta_2(\theta_0, h) + \delta_3(\theta_0, h, \beta)$$

$$\mu_2(\theta_0, h) := -86400 \cdot \left(\delta_2(\theta_0, h)\right) \qquad \boxed{\mu_2(\theta_0, h_{déc}) = -16.525}$$

$$\mu_3(\theta_0, h, \beta) := -86400 \cdot \left(\delta_3(\theta_0, h, \beta)\right) \qquad \boxed{\mu_3(\theta_0, h_{déc}, \beta_{déc}) = -25.783}$$

$$\mu_{déc}(\theta_0, h, \beta) := -86400 \cdot \left(\delta_2(\theta_0, h) + \delta_3(\theta_0, h, \beta)\right) \qquad \boxed{\mu_{déc}(\theta_0, h_{déc}, \beta_{déc}) = -42.308}$$

$$\mu_{tot}(\theta_0, h, \beta) := -86400 \cdot \left(\delta_1(\theta_0) + \delta_2(\theta_0, h) + \delta_3(\theta_0, h, \beta)\right) \qquad \boxed{\mu_{tot}(\theta_0, h_{déc}, \beta_{déc}) = -17.727}$$

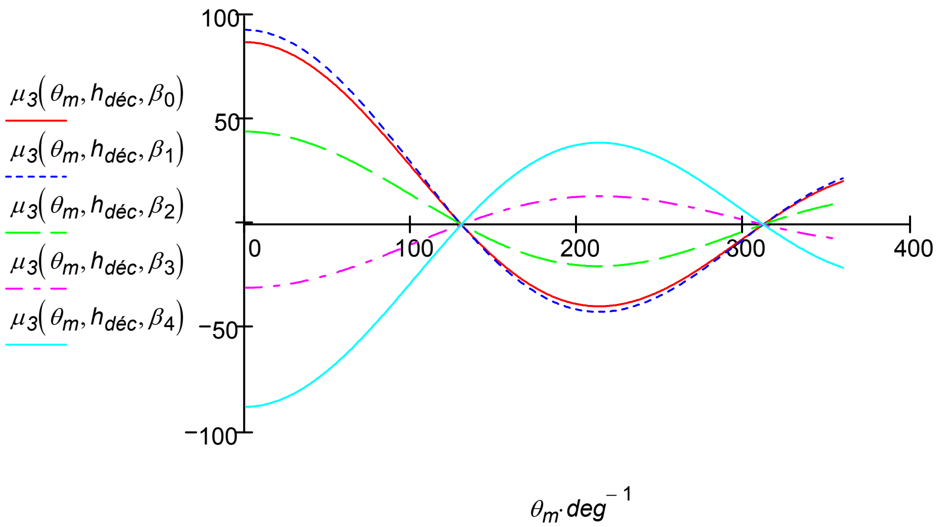


Influence de la position angulaire du décentrage initial

$i := 0, 1 .. 4 \qquad \beta_i := 45 \cdot deg \cdot i$

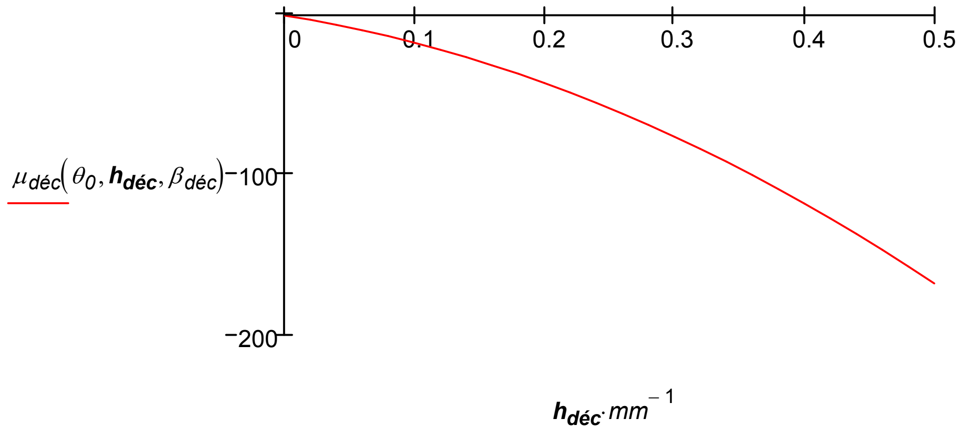
$h_{déc} = 0.2\, mm$

$\theta_0 = 270\, deg$



Influence de la position radiale du décentrage initial

$\mathbf{h_{déc}} := 0\cdot mm, .02\cdot mm .. 0.5mm$ $\qquad \beta_{déc} = 20\,deg$ $\qquad \theta_0 = 270\,deg$



Vérification par calcul numérique

$\theta(\varphi) := \theta_0\cdot cos(\varphi)$ $\qquad \sigma := R_0$ $\qquad \delta_1(\theta_0) = -2.845\times 10^{-4}$

$$\Delta_1(\theta) := \frac{i\cdot\theta}{L}\cdot R_0^{\,2}\cdot\int_0^{\psi_0} exp\left[i\cdot\left(\theta\cdot\frac{R_0}{L}+1\right)\cdot\alpha\right]d\alpha \qquad \Delta_1(\theta) := \theta\cdot\frac{R_0}{\psi_0+\theta}\cdot\left[exp\left[i\cdot\left(\psi_0+\theta\right)\right]-1\right]$$

$$\Delta_2(\theta,h,\beta) := i\cdot\frac{\theta}{L}\cdot\int_0^{L} h\cdot exp(i\cdot\beta)\cdot exp\left(i\cdot\theta\cdot\frac{s}{L}\right)ds \qquad \Delta_2(\theta,h,\beta) := -h\cdot exp(i\cdot\beta)\cdot(1-exp(i\cdot\theta))$$

$$\Delta(\theta) := \Delta_1(\theta) + \Delta_2(\theta, h_{déc}, \beta_{déc})$$

$$\chi(\theta) := \frac{\Delta(\theta)\cdot\overline{\Delta(\theta)}}{\sigma^2} \qquad \chi(\theta_0) = 9.784\times 10^{-3} \qquad \gamma(\theta) := \frac{d}{d\theta}\chi(\theta) \qquad \Gamma(\varphi) := \gamma(\theta(\varphi))$$

$$\delta_{num} := \frac{-1}{2\cdot\pi\cdot\theta_0^{\,2}}\cdot\int_0^{2\cdot\pi}\theta(\varphi)\cdot\Gamma(\varphi)\,d\varphi \qquad \boxed{\delta_{num} = 2.073\times 10^{-4}} \qquad \delta_{tot}(\theta_0, h_{déc}, \beta_{déc}) = 2.052\times 10^{-4}$$

$$\mu_{num} := -86400\cdot\delta_{num} \qquad \boxed{\mu_{num} = -17.913} \qquad \mu_{tot}(\theta_0, h_{déc}, \beta_{déc}) = -17.727$$